MATH 155 **Name**
Prep/Review for Chapter 9 Test

Use Algebraic Notation AND Show All of Your Work

For our Chapter 9 Test, you should be able to state each of the following convergence/divergence tests, or theorems:

(a)-Convergence of a Geometric Series
(b)-*n*th Term Test for Divergence
(c)-The Integral Test
(d)-Convergence of *p*-Series
(e)-Direct Comparison Test
(f)-Limit Comparison Test
(g)-Alternating Series Test
(h)-Ratio Test
(i)-Root Test

1. Determine the convergence or divergence of the following sequences:

(a) $\left\{\dfrac{n^3}{3^n}\right\}$, (b) $\left\{\dfrac{3n^2-n+4}{2n^2+1}\right\}$, (c) $\left\{\dfrac{(n+1)!}{n!}\right\}$, and (d) $\left\{\dfrac{(-1)^n}{n!}\right\}$.

If the sequence converges, find its limit. If it diverges, so state and EXPLAIN. *(Be careful with your notation, and show your steps clearly.)*

(a) $\left\{\dfrac{n^3}{3^n}\right\}$

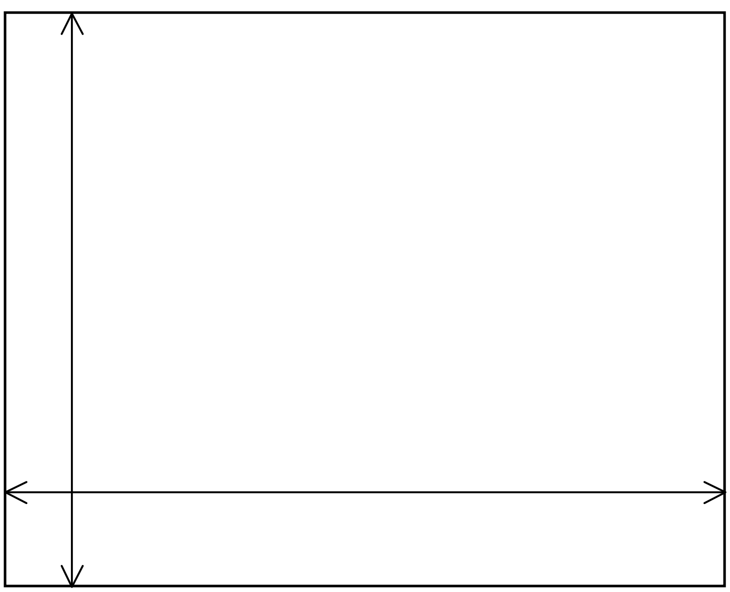
(b) $\left\{\dfrac{3n^2 - n + 4}{2n^2 + 1}\right\}$

(c) $\left\{\dfrac{(n+1)!}{n!}\right\}$

(d) $\left\{\dfrac{(-1)^n}{n!}\right\}$

2. **(a)** Find the sum of the series $\sum_{n=0}^{\infty} \frac{17}{3}\left(-\frac{8}{9}\right)^n$. **(b)** Use a **graphing utility** to find the indicated partial sum S_n and complete the table below. **(c)** Use a graphing utility to graph the first 6 terms of the sequence of partial sums. ***(Label each point.)*** **(d)** Graph the <u>horizontal line</u> that represents this the **sum** of the series and state its equation. *(Be careful with your notation, and show your steps clearly. Round any approximations to the nearest thousandths place.)*

n	**0**	**1**	**2**	**3**	**5**	**10**	**20**
S_n							

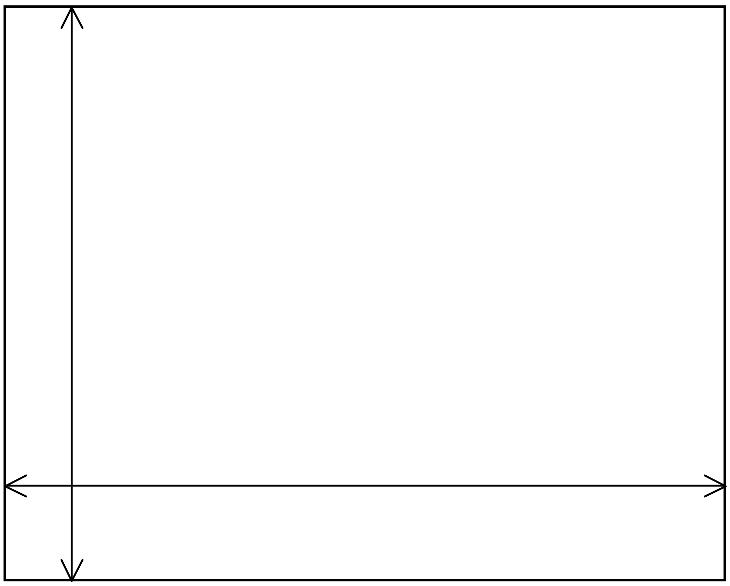


(d) Equation of Horizontal Line: ______________________________

(a) $\sum_{n=0}^{\infty} \frac{17}{3}\left(-\frac{8}{9}\right)^n =$ ______________________________

3. **(a)** Find the sum of the series $\sum_{n=1}^{\infty} \frac{6}{n(n+3)}$. **(b)** Use a **graphing utility** to find the indicated partial sum S_n and complete the table below. **(c)** Use a graphing utility to graph the first 6 terms of the sequence of partial sums. ***(Label each point.)*** **(d)** Graph the horizontal line that represents this the **sum** of the series and state its equation. *(Be careful with your notation, and show your steps clearly. Round any approximations to the nearest thousandths place.)*

n	**1**	**5**	**10**	**20**	**50**
S_n					



(d) Equation of Horizontal Line:

(a) $\sum_{n=1}^{\infty} \frac{6}{n(n+3)} =$

4. State the ***Integral Test***. Use the ***Integral Test*** to determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{\arctan(n)}{n^2+1}$. Use a graphing utility to graph $f(x)$, and verify that this graph corresponds with your result from the ***Integral Test***.
(Be careful with your notation, and show your steps clearly.)

(b) Let $f(x) =$

(c) ***three*** conditions for $f(x)$:

(a) TEST:

5. Use the ***Direct Comparison Test*** to determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{4^n}{3^n - 1}$. Be sure to show that any conditions for the application of this test are met.

(Be careful with your notation, and show your steps clearly.)

State the ***Direct Comparison Test***:

TEST:

6. Use the ***Limit Comparison Test*** to determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{n^2+10}{4n^5+n^3}$. Be sure to show that any conditions for the application of this test are met.

(Be careful with your notation, and show your steps clearly.)

State the ***Limit Comparison Test***:

TEST:

7. **(a)** Use the ***Alternating Series Test*** to prove the convergence of the series $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{3n+2}$. Be sure to show that any conditions for the application of this test are met. *(Be careful with your notation, and show your steps clearly.)*

State the ***Alternating Series Test***:

7(a) TEST:

7(b) Does $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{3n+2}$ converge absolutely, or conditionally? *(Use the definition of absolute convergence and show your reasoning clearly.)*

8. **(a)** Use the Alternating Series Remainder to determine the number of terms required to approximate the sum of $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n n!}$ with an error of less than 0.001.

(b) Use a graphing utility and your result from part **(a)** to write a **finite sum** that approximates the infinite sum, $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n n!}$, with an error of less than 0.001. *(Hint: Write out your sum and show at least **two steps** as you compute this finite approximation.)*

$\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n n!} \approx$ ______________________________

9. **(a)** Use the Alternating Series Remainder to determine the number of terms required to approximate the sum of $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n4^n}$ with an error of less than 0.001.

(b) Use a graphing utility and your result from part **(a)** to write a **finite sum** that approximates the infinite sum, $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n4^n}$, with an error of less than 0.001. *(Hint: Write out your sum and show at least **two steps** as you compute this finite approximation.)*

$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n4^n} \approx$ ____________________

10. Use the ***Ratio Test*** to determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{n!}{n3^n}$.

Be sure to show that any conditions for the application of this test are met.
(Be careful with your notation, and show your steps clearly.)

State the ***Ratio Test:***

TEST:

11. Use the ***Root Test*** to determine the convergence or divergence of the series $\sum_{n=1}^{\infty}\left(\frac{4n}{5n-3}\right)^{n}$.

(Be careful with your notation, and show your steps clearly.)

<u>State the ***Root Test:***</u>

<u>**TEST:**</u>

12. State the definition of the ***nth Taylor Polynomial*** for the function f with a center at c.

13. **(a)** Find the ***Taylor Polynomial*** of degree 3 for the function $f(x)=\sqrt{x}$ with a center at $c=4$. **(b)** Find the ***Taylor Polynomial*** of degree 4 for the function $f(x)=\ln(x)$ with a center at $c=2$. *(Be careful with your notation, and show your steps clearly.)*

(a) ***Taylor Polynomial*** of degree 3, $P_3(x)=$

(b) ***Taylor Polynomial*** of degree 4, $P_4(x)=$

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14. **(a)** Find the ***Maclaurin polynomial*** of degree 4 for the function $f(x) = \frac{1}{1+x}$. **(b)** Use this polynomial from part **(a)** to approximate $f(0.1)$? **(c)** Use Taylor's Theorem and a graphing utility to obtain an upper bound for the error of the approximation. *(Be careful with your notation, and show your steps clearly.)*

(a) ***Maclaurin polynomial*** of degree 4, $P_4(x) =$

(b) $f(0.1) \approx$

(c) upper bound for the error $\approx$

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15. **(a)** Find the ***Maclaurin polynomial*** of degree 5 for the function $f(x) = \sin(x)$. **(b)** Use this polynomial from part **(a)** to approximate $\sin(0.1)$? **(c)** Use Taylor's Theorem and a graphing utility to obtain an upper bound for the error of the approximation. *(Be careful with your notation, and show your steps clearly.)*

(a) ***Maclaurin polynomial*** of degree 5, $P_5(x) =$

(b) $\sin(0.1) \approx$

(c) upper bound for the error ≈

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16. **(a)** Find the ***radius of convergence*** of the power series $\sum_{n=0}^{\infty} \frac{(x)^n}{2^n}$.

(b) Find the ***radius of convergence*** of the power series $\sum_{n=0}^{\infty} \frac{(x)^n}{n!}$.

(Be careful with your notation, and show your steps clearly.)

(a) ***radius of convergence*** of the power series $\sum_{n=0}^{\infty} \frac{(x)^n}{2^n}$ is $R =$

(b) ***radius of convergence*** of the power series $\sum_{n=0}^{\infty} \frac{(x)^n}{n!}$ is $R =$

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17. Find the ***interval of convergence*** of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n (x+1)^n}{2^n}$. When checking for convergence at the endpoints of the interval, state the series you are testing, **state the name** of your convergence test, and state your result. *(Be careful with your notation, and show your steps clearly.)*

Interval of convergence:

18. Find the ***interval of convergence*** of the power series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-5)^n}{n5^n}$. When checking for convergence at the endpoints of the interval, state the series you are testing, **state the name** of your convergence test, and state your result. *(Be careful with your notation, and show your steps clearly.)*

Interval of convergence: ______________________________

19. Given function defined by the power series $f(x)=\sum_{n=1}^{\infty}\frac{(x)^n}{n}$, find the following series: **(a)** $\int f(x)dx$ and **(b)** $f'(x)$. *(Be careful with your notation, and show your steps clearly.)*

(a) Power series for $\int f(x)dx$ =

(b) Power series for $f'(x)$ =

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20. Given function defined by the power series $\sum_{n=0}^{\infty} \frac{(x)^n}{3^n}$, find the following series: **(a)** $\int f(x)dx$ and **(b)** $f'(x)$. *(Be careful with your notation, and show your steps clearly.)*

(a) Power series for $\int f(x)dx =$

(b) Power series for $f'(x) =$

21. **(a)** Find a power series for the function $f(x) = \dfrac{1}{2x-5}$, centered at $c = -3$. **(b)** Find the ***interval of convergence*** of this power series. *(Be careful with your notation, and show your steps clearly. Hint: Do* ***not*** *check the endpoints of the interval of convergence.)*

(a) Power series for $\dfrac{1}{2x-5}$ =

(b) Interval of convergence:

22. **(a)** Find a power series for the function $f(x) = \frac{4}{3x+2}$, centered at $c = 3$. **(b)** Find the ***interval of convergence*** of this power series. *(Be careful with your notation, and show your steps clearly. Hint: Do* ***not*** *check the endpoints of the interval of convergence.)*

(a) Power series for $\frac{4}{3x+2}$ =

(b) Interval of convergence: